Multi-Factor Design

EW Example

OBJECTIVE: Evaluate the Effectiveness of the MARK 00 Missile System Against Specified Threats.

Procedure:

1) Decide what threats are to be tested
2) Decide what factors (independent variables) are to be considered
3) Decide what dependent variable (response variable) is to be measured
4) Determine how the data is to be collected (experimental design)
5) Conduct the tests and collect the data
6) Analyze the data, test for significance of factors
7) Determine a mathematical model for relating the dependent variable to the various independent variables, which are found to significantly affect the dependent variable.

BRAND X: ORIGINAL TEST PLAN:

DEPENDENT VARIABLE: Time To Acquire, TA

INDEPENDENT VARIABLES: Target Speed (S), Target Altitude (A), ECM (M), Readiness Condition (C).

THREATS:

1	SLOW, LOW, CLEAR, III
2	FAST, HIGH, CLEAR, III
3	SLOW, LOW, CLEAR, I
4	FAST, LOW, CLEAR, I
5	SLOW, HIGH, SOJ, III
6	SLOW, LOW, SOJ, I
7	FAST, LOW, SOJ, I

BRAND X: Experimental Design:

- **Run Each Threat 5 Times in the following order**:

RUNS	THREAT
1-5	1
6-10	2
11-15	3
16-20	4
21-25	5
26-30	6
31-35	7

Because of time and resource constraints, a maximum of 16 runs can be made in a single day. Plans were to make 10 runs one day, followed by 15 runs the second day, and 10 runs the third day.

COMMENTS: The original test plan was the brand X approach of one threat at a time. Unfortunately, this is the approach often seen in test plans. Only part of the data is used to address each threat. The threats can only be interrelated piecemeal. Also, there is hopeless confounding of some factors with other factors. Suppose, for example, that weather, learning, environmental conditions or some other factors affect TA; with the original test plan those factors could not be separated from the controlled factors. Thus, there would be no way to determine what effects are due to the threats and what is due to the other uncontrolled factors.

THREAT MATRIX

ECM	Alt >> RC	(-) SLOW		(+) FAST	
		(-) LOW	(+) HIGH	(-) LOW	(+) HIGH
CLEAR (-)	(+) I	3		4	
	(-) III	1			2
SOJ (+)	(+) I	6		7	
	(-) III		5		

The original 7 threats are labeled. Those are the critical ones, but the other threats are also of interest and we would like to test all 16 threats.

REVISED PLAN: Test all 16 threats replicated 2 times (32 runs total). We can do this in such a way that we can eliminate the confounding problems and we can actually address the original 7 threats with even more information than the original design with 5 replications.

Furthermore, we can determine the effects of each factor and their interactions. Does this sound impossible? This is the value of good experimental design! In the actual tests, the 16 conditions were run in random order and replicated the second day. The data were as follows:

EW Design Matrix and Data

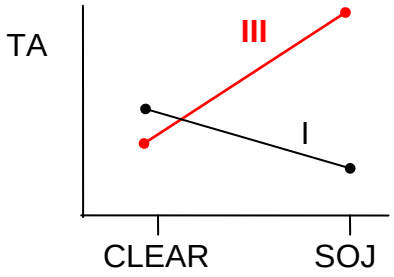
	Speed (s)	(-) SLOW		(+) FAST	
ECM (m)	Alt (a) / RC (c)	(-) LOW	(+) HIGH	(-) LOW	(+) HIGH
CLEAR (-)	(+) I	51 83	59 62	77 73	44 72
	(-) III	63 65	47 69	81 67	50 51
SOJ (+)	(+) I	45 56	58 58	86 85	39 38
	(-) III	85 80	59 77	102 135	75 91

PLOT INTERACTIONS

Plot the means of the four interaction combinations. Connect like Factor Levels. Look for non-parallel or crossing lines and military significance

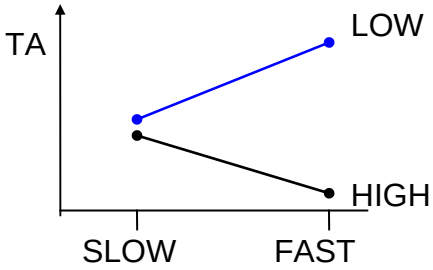
ECM x RC (mxc) INTERACTION

	I	III
CLEAR	65.13	61.63
SOJ	58.13	88.00



SpeedxAlt (sxa) INTERACTION

	LOW	HIGH
SLOW	66.00	61.13
FAST	88.25	57.50



COMMENT: In performing the analysis, say for threat I (slow, high, clear, III) we do not simply average the two values 47 and 69. Instead, we determine the effects due to slow speeds, high altitudes, SxA interaction, clear ECM, condition III and ECMxRC interaction and add them up. In estimating each of these effects we use at least 8 observations, and in most cases 16 are used. Thus, we actually base our estimates on more observations than did the original test plan. In addition, we can estimate the time to acquire for each of the 16 threats, and we discovered that two of the interactions were significant in their effect on the time to acquire. This could not have been discovered with the original design. Tactics may be affected by this discovery.

HYPOTHESIZED MATHEMATICAL MODEL:

$$TA_{ijklr} = \mu + s_i + a_j + m_k + c_l + (sa)_{ij} + (sm)_{ik} + (sc)_{il} + (am)_{jk} + (ac)_{jl} + (mc)_{kl}$$

$$+ (sam)_{ijk} + (smc)_{ikl} + (amc)_{jkl} + \rho_r + e_{ijklr}$$

An analysis of variance revealed that all 4 treatment main effects were significant, the two-way interactions (sa) and (mc) were significant and the replication effect (weather, practice) was significant. This led to the reduced model:

$$TA_{ijklr} = \mu + (s_i + a_j + (sa)_{ij}) + (m_k + c_l + (mc)_{kl}) + \rho_r + e_{ijklr}$$

ANOVA TABLE FROM MINITAB:

SOURCE	DF	SEQ SS	ADJ SS	ADJ MS	F	P
MAIN FACTORS						
SPEED (s)	1	693.8	693.8	693.8	4.92	0.041
ALT (a)	1	2538.3	2538.3	2538.3	18.01	0.001
ECM (m)	1	750.8	750.8	750.8	5.33	0.035
RC (c)	1	1391.3	1391.3	1391.3	9.87	0.006
REPL	1	621.3	621.3	621.3	4.41	0.052
INTERACTIONS						
SPEED (s)*ALT (a)	1	1339.0	1339.0	1339.0	9.50	0.007
SPEED (s)*ECM (m)	1	427.8	427.8	427.8	3.04	0.101
SPEED (s)*RC (c)	1	132.0	132.0	132.0	0.94	0.347
ALT (a)*ECM (m)	1	166.5	166.5	166.5	1.18	0.293
ALT (a)*RC (c)	1	34.0	34.0	34.0	0.24	0.630
ECM (m)*RC (c)	1	2227.8	2227.8	2227.8	15.81	0.001
SPEED (s)*REPL	1	19.5	19.5	19.5	0.14	0.715
ALT (a)*REPL	1	34.0	34.0	34.0	0.24	0.630
ECM (m)*REPL	1	0.0	0.0	0.0	0.00	0.988
RC (c)*REPL	1	0.8	0.8	0.8	0.01	0.942
RESIDUAL						
Error	16	2254.5	2254.5	140.9		
Total	31	12631.5				

ESTIMATED EFFECTS AND COEFFICIENTS FOR TA:

First Assign Factor Level Settings in Minitab:

STAT>DOE>Factorial>Define Custom Design: LOW/HIGH

To generate the below table in Minitab:

STAT>DOE>Factorial>Analyze Factorial Design: Terms; Model (2)

Term	Effect	Coef	SE Coef	T	P
Constant		68.2	2.088	32.67	0.000
SPEED (s)	9.312	4.7	2.088	2.23	0.037
ALT (a)	-17.813	-8.9	2.088	-4.27	0.000
ECM (m)	9.688	4.8	2.088	2.32	0.031
RC (c)	-13.188	-6.6	2.088	-3.16	0.005
SPEED*ALT (sa)	-12.937	-6.5	2.088	-3.10	0.005
SPEED*ECM (sm)	7.313	3.7	2.088	1.75	0.095
SPEED*RC (sc)	-4.062	-2.0	2.088	-0.97	0.342
ALT*ECM (am)	-4.563	-2.3	2.088	-1.09	0.287
ALT*RC (ac)	2.063	1.0	2.088	-0.49	0.627
ECM*RC (mc)	-16.688	-8.3	2.088	-4.00	0.001

REGRESSION MODEL COEFFICIENTS:

From ANOVA Table: Speed (s), Alt (a), ECM (m), RC (c), (sa) & (mc) are largest factor effects. Therefore a reduced regression model can be used to predict Threat Values

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{34} x_3 x_4 + e$$

Where: x_1 represents factor Speed (s). x_2 represents factor Altitude (a)
x_3 represents factor ECM (m), x_4 represents factor RC (c),
x_1x_2 represents (sa) Interaction, x_3x_4 represents (mc) Interaction

The reduced Regression Model Coefficients (not including replication effects) are estimated by one-half the corresponding factor effects. The constant is estimated by the grand average as shown below:

$$\hat{T}A = 68.2 + 4.7(s) - 8.9(a) - 6.5(sa) + 4.8(m) - 6.6(c) - 8.3(mc)$$

PREDICTED VALUES FOR EACH THREAT

The predicted value for each Threat would be obtained by substituting the appropriate average of the low or high levels for each factor and interaction or dividing the selected factor effect by two and multiplying by the sign of the level and entering them into the reduced equation.

For example, for our estimate of the time to acquire for Threat 2: Speed Fast (+), Alt Hi (+), ECM Clear (-), and RC III (-), we obtain the following:

$$\mu = 68.2$$

$$(+s_{fast} + a_{hi} + (sa)_{fast/hi}) = 4.7 + (-8.9) + (-6.5) = -10.7$$

$$(+ m_{clr} + c_{III} + (mc)_{clr/III})) = + (-1)(4.8) + (-1)(-6.6) + (-)(-)(-8.3) = -6.5$$

Combining, we get for Threat 2 an estimate of:

$$TA_{threat\ 2} = 68.2 + (-10.7) + (-6.5) = 51.0 \text{ secs.}$$

This can be repeated for all 16 threats.

Note: Since there was a significant replication effect, I would examine all the background variables to determine what the potential causes of the replication effect were and how it would affect the resolution of the Critical Operational Issues.

RESIDUALS

Since the observed values of the two runs for Threat 2 were 50 and 51, the residuals are 50 – 68.2 = -18.2 and 51 – 68.2 = -17.2. Residuals for the other 30 runs are obtained similarly. A normal probability plot of the residuals can then be drawn to determine if any non-normality exists.